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Bending stress and shear stress in beams

Beams are one of the main design elements a structural engineer will work with. This is a loaded simply supported beam: For the PE exam, a loaded beam has two main options for failure, shear and bending. Shear Stress Shear stress is caused by forces acting perpendicular to the beam. Shear forces are visible in both cross sections and profiles. The formula for average shear at a spot on a beam is: F is the force applied (from the shear diagram or by inspection) A is the cross-sectional area of the beam. When looking at the shear load dispersed throughout a cross-section the load is highest at the middle and tapers off to the top and bottom. The formula for max shear in a few different shapes is: For I-Beams the shear is generally only considered in the web of the beam. The web is the long vertical part. These ares are all listed in the Steel Manual and may also be in some other more general test references. Remember to use the maximum shear force (found from a shear diagram or by inspection) when finding the maximum shear. The maximum shear in the simply supported beam pictured above will occur at either of the reactions. Bending Stress (Stress from Moments) Loads on a beam result in moments which result in bending stress. Loaded simply supported beams (beams supported at both ends like at the top of the article) are in compression along the top of the member and in tension along the bottom, they bend in a "smile" shape. Here is a cross section at an arbitrary spot in a simply supported beam: Loaded Cantilever beams (beams mounted on one end and free on the other) are in tension along the top and compression along the bottom. These would bend downward in a "half frown". For these the picture above would be upside down (tension on top etc). Each beam and loading configuration is different, and even segments differ within the same beam! The formula to determine bending stress in a beam is: Where M is the moment at the desired location for analysis (from a moment diagram). c is the distance from the neutral axis to the outermost section (for symmetric cross sections this is half the overall height but for un-symmetric shapes the neutral axis is not at the midpoint). I is the Moment of Inertia. For a rectangular beam . I is also given in tables in the steel manual and other reference materials. The moment of inertia and c are often combined into a single number representing the physical characteristics of the cross-section, S. Use S like so: S is given in many tables and can save a lot of time on the exam. For a rectangular beam . The maximum stress for a beam uses the same formula as above but make sure to use the highest moment in the member, this is found on the moment diagram. Notes Bending Stress is higher than Shear stress in most cases. If the member is really short or there is a high load close to a support (cutting the beam like scissors) then the shear force may govern. Watch out for those cases. Unit 6: Bending and shear Stresses in beams Upcoming SlideShare Loading in …5 × 1 Like this document? Why not share! 1. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 1 TABLE OF CONTENTS 6.1. INTRODUCTION .... 2 6.2. SIMPLE BENDING .... 2 6.3. ASSUMPTIONS IN SIMPLE BENDING .... 3 6.4. DERIVATION OF BENDING EQUATION.. 3 6.5. SECTION MODULUS.. 5 6.6. MOMENT CARRYING CAPACITY OF A SECTION .... 6 6.7. SHEARING STRESSES IN BEAMS.. 8 6.8 SHEAR STRESSES ACROSS RECTANGULAR SECTIONS .... 9 WORKED EXAMPLES .... 11 REFERENCES:.. 15 2. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 2 UNIT-6 BENDING AND SHEAR STRESSES IN BEAMS Syllabus Introduction, Theory of simple bending, assumptions in simple bending, Bending stress equation, relationship between bending stress, radius of curvature, relationship between bending moment and radius of curvature, Moment carrying capacity of a section. Shearing stresses in beams, shear stress across rectangular, circular, symmetrical I and T sections. (composite / notched beams not included). 6.1. INTRODUCTION When some external load acts on a beam, the shear force and bending moments are set up at all sections of the beam. Due to the shear force and bending moment, the beam undergoes certain deformation. The material of the beam will offer resistance or stresses against these deformations. These stresses with certain assumptions can be calculated. The stresses introduced by bending moment are known as bending stresses. In this chapter, the theory of pure bending, expression for bending stresses, bending stress in symmetrical and unsymmetrical sections, strength of a beam and composite beams will be discussed. E.g., Consider a piece of rubber, most conveniently of rectangular cross-section, is bent between one's fingers it is readily apparent that one surface of the rubber is stretched, i.e. put into tension, and the opposite surface is compressed. 6.2. SIMPLE BENDING A theory which deals with finding stresses at a section due to pure moment is called bending theory. If we now consider a beam initially unstressed and subjected to a constant B.M. along its length, it will bend to a radius R as shown in Fig. b. As a result of this bending the top fibres of the beam will be subjected to tension and the bottom to compression. Somewhere between the two surfaces, there are points at which the stress is zero. The locus of all such points is termed the neutral axis (N.A). The radius of curvature R is then measured to this axis. For symmetrical sections the N.A. is the axis of symmetry, but whatever the section the N.A. will always pass through the centre of area or centroid. 3. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 3 Beam subjected to pure bending (a) before, and (b) after, the moment M has been applied. In simple bending the plane of transverse loads and the centroidal plane coincide. The theory of simple bending was developed by Galelio, Bernoulli and St. Venant. Sometimes this theory is called Bernoulli's theory of simple bending. 6.3. ASSUMPTIONS IN SIMPLE BENDING The following assumptions are made in the theory of simple bending: 1 The beam is initially straight and unstressed. 2 The material of the beam is perfectly homogeneous and isotropic, i.e. of the same density and elastic properties throughout. 3 The elastic limit is nowhere exceeded. 4 Young's modulus for the material is the same in tension and compression. 5 Plane cross-sections remain plane before and after bending. 6 Every cross-section of the beam is symmetrical about the plane of bending, i.e. about an axis perpendicular to the N.A. 7 There is no resultant force perpendicular to any cross-section. 8 The radius of curvature is large compared to depth of beam. 6.4. DERIVATION OF BENDING EQUATION Consider a length of beam under the action of a bending moment M as shown in Fig. 6.2a. N-N is the original length considered of the beam. The neutral surface is a plane through X-X. In the side view NA indicates the neutral axis. O is the centre of curvature on bending (Fig. 6.2b). 4. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 4 Fig. 6.2 Let R = radius of curvature of the neutral surface = angle subtended by the beam length at centre O = longitudinal stress A filament of original length NN at a distance v from the neutral axis will be elongated to a length AB ..(i) Thus stress is proportional to the distance from the neutral axis NA. This suggests that for the sake of weight reduction and economy, it is always advisable to make the cross-section of beams such that most of the material is concentrated at the greatest distance from the neutral axis. Thus there is universal adoption of the I-section for steel beams. Now let A be an element of cross-sectional area of a transverse plane at a distance v from the neutral axis NA (Fig. 6.2). For pure bending, Net normal force on the cross-section = 0 5. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 5 This indicates the condition that the neutral axis passes through the centroid of the section. Also, bending moment = moment of the normal forces about neutral axis Or R E I M (ii) Where dAyI 2 and is known as the moment of inertia or second moment of area of the section. From (i) and (ii), yR E I M   Where, M = Bending Moment at a section (N-mm). I = Moment of Inertia of the cross section of the beam about Neutral axis (mm4). = Bending stress in a fibre located at distance y from neutral axis (N/mm2). This stress could be compressive stress or tensile stress depending on the location of the fibre. y = Distance of the fibre under consideration from neutral axis (mm). E = Young's Modulus of the material of the beam (N/mm2). R = Radius of curvature of the bent beam (mm). 6.5. SECTION MODULUS The maximum tensile and compressive stresses in the beam occur at points located farthest from the neutral axis. Let us denote y1 and y2 as the distances from the neutral axis to the extreme fibres at the top and the bottom of the beam. Then the maximum bending normal stresses are t bc Z M yI M I My 1 1 , bc is bending compressive stress in the topmost layer. Similarly, b bt Z M yI M I My 2 2 , bt is bending compressive stress in the topmost layer. Here, Zt and Zb are called section moduli of the cross sectional area, and they have dimensions of length to the third power (ex. mm3). If the cross section is symmetrical (like rectangular or square sections), then Zt = Zb = Z, and Z is called as section modulus. Section modulus is defined 6. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 6 as the ratio of rectangular moment of inertia of the section to the distance of the remote layer from the neutral axis. Thus, general expression for bending stress reduces to Z M It is seen from the above expression that for a given bending moment, it is in the best interests of the designer of the beam to procure high value for section modulus so as to minimise the bending stress. More the section modulus designer provides for the beam, less will be the bending stress generated for a given value of bending moment. 6.6. MOMENT CARRYING CAPACITY OF A SECTION From bending equation we have I My It shows bending stress is maximum on the extreme fibre where y is maximum. In any design this extreme fibre stress should not exceed maximum permissible stress. If per is the permissible stress, then in a design per max pery I M Or if M is taken as maximum moment carrying capacity of the section, pery I M max Or per y I M max The moment of inertia I and extreme fibre distance ymax are the properties of cross-section. Hence, I/ymax is the property of cross-sectional area and is termed as section modulus and is denoted by Z. Thus the moment carrying capacity of a section is given by ZM per If permissible stresses in tension and compression are different, moment carrying capacity in tension and compression are found separately by considering respective extreme fibres and the smallest one is taken as moment carrying capacity of the section. 7. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 7 Expressions for section modulus of various standard cross-sections are derived below. Rectangular section of width b and depth d: Hollow rectangular section with symmetrically placed opening: Consider the section of size B x D with symmetrical opening bx d as shown in Fig.. Circular section of diameter d Hollow circular section of uniform thickness: 8. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 8 Triangular Section 6.7. SHEARING STRESSES IN BEAMS we know that beams are usually subjected to varying bending moment and shearing forces. The relation between bending moment M and shearing force F is dM/dx=F. Bending stress act longitudinally and its intensity is directly proportional to its distance from neutral axis. Now we will find the stresses induced by shearing force. Consider an elemental length of beam between the sections A-A and B- B separated by a distance dx as shown in Fig. 6.3a. Let the moments acting at A- A and B-B be M and M+dM. Fig. 6.3 9. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 9 Let CD be a fibre at a distance y-from neutral axis. Then bending stress at left side of the element = y I M The force on the element on left side = ybdy I M Similarly due to bending, force on the right side of the element = ybdy I dMM Unbalanced force towards right in element = ybdy I dM ybdy I M ybdy I dMM   There are a number of such elements above section CD. Hence unbalance horizontal force above section CD = ' y y ybdy I dM This horizontal force is resisted by shearing stresses acting horizontally on plane at CD. Let intensity of shearing stress be q. Then equating shearing force to unbalanced horizontal force we get = ' y y ybdy I dM bdx Or ' 1 y y ya bIdx dM Where a = b dy is area of element. The term ' y y ya can be looked as yaay y y ' Where ya is the moment of area above the section under consideration about neutral axis. From equation, dM/dx=F ya bI F From the above expression it may be noted that shearing stress on extreme fibre is zero. 6.8 SHEAR STRESSES ACROSS RECTANGULAR SECTIONS Consider a rectangular section of width b and depth d subjected to shearing force F. Let A-A be the section at distance y from neutral axis as shown in Fig. 6.4. We know that shear stress at this section. ya bI F 10. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 10 Fig. 6.4 where ya is the moment of area above this section (shown shaded) about the neutral axis. i.e., shear stress varies parabolically. When y=d/2, y=d/2, y = 0, is maximum and its value is 4 6 2 3max d bd F avg bd F 5.15.1 Where bd F Area rceShearingFo avg Thus, maximum shear stress is 1.5 times the average shear stress in rectangular section and occurs at the neutral axis. Shear stress variation is parabolic. Shear stress variation diagram across the section is shown in Fig.6.4b. 11. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 11 WORKED EXAMPLES 1) A simply supported beam of span 5 m has a cross-section 150 mm * 250 mm. If the permissible stress is 10 N/mm2 , find (a) maximum intensity of uniformly distributed load it can carry. (b) maximum concentrated load P applied at 2 m from one end it can carry. Solution: Moment carrying capacity M = Z = l0 x 1562500 N - mm (a) If w is the intensity of load in N/m units, then maximum moment Equating it to moment carrying capacity, we get maximum intensity of load as (b) If P is the concentrated load as shown in Fig., then maximum moment occurs under the load and its value 2) A symmetric I-section has flanges of size 180 mm x 10 mm and its overall depth is 500 mm. Thickness of web is 8 mm. It is strengthened with a plate of size 240 mm x 12 mm on compression side. Find the moment of resistance of the section, if permissible stress is 150 N/mm2 . How much uniformly distributed load it can carry if it is used as a cantilever of span 3 m? Solution The section of beam is as shown in Fig. Let y be the distance of centroid from the bottom-most fibre. 12. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 12 Moment of resistance (Moment carrying capacity) 13. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 13 Let the load on cantilever be w/m length as shown in Fig. 3) A T-section is formed by cutting the bottom flange of an I-section. The flange is 100 mm x 20 mm and the web is 150 mm x 20 mm. Draw the bending stress distribution diagrams if bending moment at a section of the beam is 10 kN-m (hogging). Solution M = 10 kN-m = 10 x 106 N mm (hogging) Maximum bending stresses occur at extreme fibres, i.e. at the top bottom fibres which can be computed as I My (i) Moment of inertia is given by Substituting these values in Eq. (1), Stress in the top fibre = Stress in the bottom fibre = 14. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 14 The given bending moment is hogging and hence negative and the tensile stresses occur at top fibre and compressive stresses in bottom fibres. 4) Fig. shows the cross-section of a beam which is subjected to a shear force of 20 kN. Draw shear stress distribution across the depth marking values at salient points. Solution Let y, be the distance of C.G form top fibre. Then taking moment of area about top fibre and dividing it by total area, we get 15. Mechanics of Materials 10ME34 Compiled by Hareesha N G, Asst Prof, DSCE Page 15 ya bI F REFERENCES: 1) A Textbook of Strength of Materials By R. K. Bansal 2) Fundamentals Of Strength Of Materials By P. N. Chandramouli 3) Strength of Materials By B K Sarkar 4) Strength of Materials S S Bhavikatti 5) Textbook of Mechanics of Materials by Prakash M. N. Shesha, suresh G. S. 6) Mechanics of Materials: with programs in C by M. A. Jayaram